Exam 1, Calculus III
Math 2210, Spring 2010, Dr. Armstrong Name ______________________

Instructions: Show all your work wherever required if you hope for some partial credit. Note the formula sheet on the last page; you may detach it for use and discard it when you're done if you like. Calculators are allowed except where specifically prohibited.

1. Let points $P = (2, 1, -1)$, $Q = (0, 1, 4)$, and $R = (3, 3, 3)$. (Pts: a:4,b:4,c:5,d:6,e:6)

   (a) Find vectors $\mathbf{u}$ and $\mathbf{v}$ in standard position such that $\mathbf{u} = \overrightarrow{PQ}$ and $\mathbf{v} = \overrightarrow{PR}$. (4 pts)

   (b) Use the dot product of $\mathbf{u}$ and $\mathbf{v}$ to tell whether the angle between them is acute, obtuse, or right. (4 pts)

   (c) Find a parametric representation for the line through P and Q. (5 pts)

   (d) Find the area of triangle ΔPQR. (6 pts)

   (e) Find an equation for the plane containing P, Q, and R. (4 pts)

2. Find the torque (in Joules, or N-m) generated by the force $\mathbf{F}$ on the wrench as diagrammed below. (8 pts)

3. Find an equation for a line that contains the point (3,1,-2) and that is (a) perpendicular to (b) parallel to the plane $2x - z = 5$. (5 pts each)

   (a)

   (b) (Note: There are infinitely many lines parallel to a plane through a given point. Find any one of them.)

4. Find any and all points where the line $\mathbf{r}(t) = (2-t)\,\mathbf{i} + t\,\mathbf{j} + (1+t)\,\mathbf{k}$ intersects the sphere $x^2 + y^2 + z^2 = 5$. (8 pts)

5. Sketch a graph of the quadric surface. Show clearly how you work with traces. No calculator is allowed here. (9 pts)

$$y^2 = x^2 + \frac{z^2}{4}$$

6. Sketch a graph of each equation. If the graph you draw looks rough, you can help your case by describing what you're doing. No calculator is allowed here. (Pts: a:5,b:5,c:6)

   (a) $x^2 + y^2 = 4$

   (b) The spherical coordinate equation $\phi = \pi/4$.

   (c) The curve (including direction) defined by $\mathbf{F}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \left(\frac{t}{\pi}\right)\mathbf{k}$, $0 \le t \le 4\pi$.

7. Convert each point or equation to the other coordinate system as requested, simplifying the equation as much as can be done. Remember that we consider only nonnegative r and ρ values.

   (a) The rectangular point $(1, -\sqrt{3}, -2)_R$ into spherical coordinates (6 pts)

   (b) The cylindrical equation $r^2 + z^2 = 2r\cos\theta$ into spherical coordinates (5 pts)

   (c) The cylindrical equation $r = 4\sin\theta$ into rectangular coordinates (5 pts)

8. Define the vector-valued function $\mathbf{F}$ by

$$\mathbf{F}(t) = 2t\,\mathbf{i} + e^{2t}\,\mathbf{j} + \frac{3}{t+2}\,\mathbf{k}.$$

   (a) Evaluate $\mathbf{F}'(t)$. (5 pts)

   (b) Evaluate $\int \mathbf{F}(t)\,dt$. (5 pts)

Formulas

- Resultant of two forces: $||\mathbf{F}_1 + \mathbf{F}_2||^2 = ||\mathbf{F}_1||^2 + ||\mathbf{F}_2||^2 + 2||\mathbf{F}_1||\,||\mathbf{F}_2||\cos\phi$
- For the angle α between $\mathbf{F}_1$ and the resultant force: $\sin\alpha = \dfrac{||\mathbf{F}_2||}{||\mathbf{F}_1 + \mathbf{F}_2||}\sin\phi$
- Angle θ between vectors: $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{||\mathbf{u}||\,||\mathbf{v}||}$, or $\sin\theta = \dfrac{||\mathbf{u}\times\mathbf{v}||}{||\mathbf{u}||\,||\mathbf{v}||}$ for $0 \le \theta \le 90°$
- Projection of $\mathbf{v}$ onto $\mathbf{b}$: $\text{proj}_{\mathbf{b}}\mathbf{v} = \left(\dfrac{\mathbf{v}\cdot\mathbf{b}}{||\mathbf{b}||^2}\right)\mathbf{b}$
- Work from P to Q with force $\mathbf{F}$: $W = \mathbf{F}\cdot\overrightarrow{PQ} = ||\mathbf{F}||\,||\overrightarrow{PQ}||\cos\theta$
- Area of a parallelogram $A = ||\mathbf{u}\times\mathbf{v}||$
- Volume of a parallelepiped $V = |\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w})|$
- Torque or scalar moment of $\mathbf{F}$ at P: $T = ||\overrightarrow{PQ}\times\mathbf{F}|| = ||\overrightarrow{PQ}||\,||\mathbf{F}||\sin\theta$
- Distance between a point $P(x_0, y_0, z_0)$ and a plane $D = \dfrac{|ax_0 + by_0 + cz_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$
- Cylindrical coordinates: $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$, $\begin{cases} r = \sqrt{x^2 + y^2} \\ \tan\theta = y/x \\ z = z \end{cases}$
- Spherical coordinates: $\begin{cases} x = \rho\sin\phi\cos\theta \\ y = \rho\sin\phi\sin\theta \\ z = \rho\cos\phi \end{cases}$, $\begin{cases} \rho = \sqrt{x^2 + y^2 + z^2} \\ \tan\theta = y/x \\ \cos\phi = z/\sqrt{x^2 + y^2 + z^2} \end{cases}$