Practice exercises - Exam 4 (Sections 10.6 through 10.10 and 11.1 through 11.3)

1. Use the alternate series test to conclude that the series $\sum_{k=2}^{\infty}(-1)^k \frac{1}{\ln k}$ converges.

2. Determine whether the infinite series $\sum_{k=2}^{\infty}(-1)^k \cdot \frac{2^k}{k!}$ converges absolutely, converges conditionally, or diverges.

3. Find the fourth order Maclaurin polynomial for $f(x) = \sqrt{1-x}$

4. Find the Taylor series of $f(x) = \ln x$ based at the point $a = e$

5. Looking for familiar Maclaurin series, $1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \cdots$ represents what function?

6. Remembering that $\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots$, differentiate twice to find the Maclaurin series for $\frac{1}{(1-x)^3}$

7. Find the radius of convergence for the power series $\sum_{k=1}^{\infty}\left(k \cdot \pi^k\right) x^k$

8. Find the interval of convergence for the series $\sum_{k=1}^{\infty} \frac{1}{k \cdot 2^k}(x-1)^k$

9. Calculate the slope of the tangent to the parametric curve $x = \ln t$ and $y = 2 - t^2$ at $t = 1$

10. Convert

    (a) $(r, \theta) = (2, \frac{\pi}{6})$ to rectangular coordinates

    (b) $(x, y) = (-2, 2)$ to polar coordinates.

11. Using your calculator, sketch the graph of the polar curve $r = 2\sin(\theta)\tan(\theta)$

12. Find the length of the polar curve $r = \sin(\theta) + \cos(\theta)$ for $0 \leq \theta \leq \pi$

13. Find the area inside one leaf of the rose $r = \cos(3\theta)$.

Answers

1. Notice that $\lim_{k\to\infty} \frac{1}{\ln k} = 0$ and, for $f(x) = \frac{1}{\ln x}$, $f'(x) = -1 \cdot (\ln x)^{-2} \cdot \frac{1}{x} < 0$ for $x \geq 2$.
   The terms go to 0 and are decreasing, so it convergences by the alternating series test.

2. The series converges absolutely.
   Applying the ratio test to the series with absolute values gives a $\lim_{k\to\infty} \frac{2}{k+1} = 0 < 1$.

3. After differentiating and evaluating at zero, we get $M_4(x) = 1 - \frac{1}{2}x - \frac{1}{4}\frac{x^2}{2!} - \frac{3}{8}\frac{x^3}{3!} - \frac{15}{16}\frac{x^4}{4!}$

4. After differentiating and evaluating at e, we get
   $1 + \frac{1}{e}(x-e) - \frac{1}{2e^2}(x-e)^2 + \frac{1}{3e^3}(x-e)^3 - \frac{1}{4e^4}(x-e)^4 + \frac{1}{5e^5}(x-e)^5 + \cdots$

5. x^2 has been substituting for x in the Maclaurin series for e^x, so this represents e^{x^2}.

6. Differentiating the series for $(1-x)^{-1}$ twice gives
   $2(1-x)^{-3} = 2 + 6x + 12x^2 + 20x^3 + 30x^4 + \cdots$
   Dividing both sides by two gives $\frac{1}{(1-x)^3} = 1 + 3x + 6x^2 + 10x^3 + 15x^4 + \cdots$

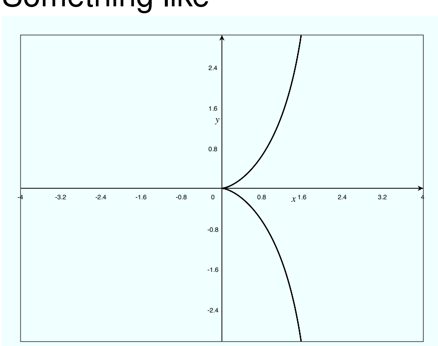
7. The ratio test on the absolute values gives $\pi\,|x| < 1$ so the radius of convergence is $\frac{1}{\pi}$

8. The ratio test on the absolute values gives $\frac{|x-1|}{2} < 1$ so that $-1 < x < 3$. Testing the endpoints gives the interval of convergence $[-1, 3)$

9. $\frac{dy}{dx} = \frac{-2t}{(1/t)} = -2t^2$ so at $t = 1$ we get $\frac{dy}{dx} = -2$

10. (a) $(\sqrt{3}, 1)$ and (b) $(2\sqrt{2}, \frac{3\pi}{4})$

11. Something like



12. $\int_0^\pi \sqrt{(f(\theta))^2 + (f'(\theta))^2}\,d\theta$ simplifies to $\int_0^\pi \sqrt{2}\,d\theta$ which is $\sqrt{2}\pi$

13. $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{1}{2}(\cos(3\theta))^2\,d\theta = \frac{1}{4}\theta + \frac{1}{24}\sin(6\theta)$ evaluated between $-\frac{\pi}{6}$ and $\frac{\pi}{6}$. This gives area $\frac{\pi}{12}$