Practice exercises - Exam 4 (Chapter 5)

1. Illustrate how to calculate the difference $-3 - 5$ using the charge model.
2. Illustrate the sum $4 + -9$ using the number line model.
3. Calculate the value of the following expressions:

   (a) $-2 + 5(7 - -3)$

   (b) $4 - 7 \cdot -2 + (3 - 8)$

4. Use the definition of division to rewrite the quotient $45 \div -9$ as a product. Use this product to find the value of the quotient.
5. Find all integer values of x (if any exist) that make the following statements true:

   (a) $x \div -3 = -2$

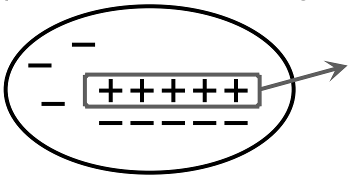
   (b) $x \div 0 = 5$

   (c) $x \cdot x = 9$

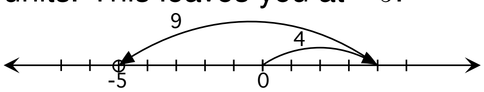
6. Find all values of the digit $\square$ so that the number $23,15\square$ is both a multiple of 3 and a multiple of 2.
7. Suppose c is a composite number and a and b are natural numbers. If c divides the product ab, can we conclude that $c|a$ or $c|b$? Justify your answer.
8. Determine whether 281 is prime or composite.
9. Construct two different factor trees for 126.
10. Using prime factorizations, find the LCM$(14, 44)$ and the GCD$(14, 44)$.
11. Listing out all divisors, find GCD$(16, 35)$.
12. Listing out multiples, find LCM$(10, 12)$.
13. Suppose the GCD$(a, b) = 15$ and the product $a \cdot b = 390$. Find the LCM(a, b).

Answers

1. Start with 3 negatives. Because this is a subtraction problem, we need to take away 5 positives. Insert positive-negative pairs until you have 5 positives. Taking away these 5 positives leaves 8 negatives or -8



2. Starting at 0, move 4 units right. Since this is addition, face right, but go backwards 9 units. This leaves you at -5:



3. (a) 48 and (b) 13

4. Suppose $45 \div -9 = \square$, then by definition $45 = -9 \times \square$. We know that 9×5 is 45, but we must have two negatives to multiply and give us 45, thus $\square = -5$

5. (a) 6 ; (b) no solution, division by zero is not defined (why?) ; and (c) $x = 3$ or $x = -3$

6. Using divisibility rules, $\square$ must be 4. In particular, to be a multiple of 2 we must have $\square$ even, and to be a multiple of 3 we need $\square$ to be either 1, 4, or 7 to get the digits adding to a multiple of 3.

7. No, consider the composite number $c = 6$ and let $a = 4$ and $b = 3$ be the two natural numbers. Then $c = 6$ divides the product $a \cdot b = 12$, but $c = 6$ does not divide either $a = 4$ or $b = 3$.

8. Remember that we only have to test for divisibility by primes up to the square root of the number. In this case, $\sqrt{281} \approx 16.76$ so we only need to test for divisibility of 2, 3, 5, 7, 11, and 13. None of these are factors of $281,$ so 281 is prime.

9. There are many different ways to construct the tree. For instance, we could factor into $2 \cdot 63$ first, then split the 63 into $7 \cdot 9$ and finally the 9 into $3 \cdot 3$. Alternately, we could factor into $3 \cdot 42$ first, then split the 42 into $6 \cdot 7$, and finally the 6 into $2 \cdot 3$. Either way, $126 = 2 \cdot 3^2 \cdot 7$.

10. $14 = 7 \cdot 2$ and $44 = 2^2 \cdot 11$ so that LCM$(14, 44) = 2^2 \cdot 7 \cdot 11 = 308$ and GCD$(14, 44) = 2$.

11. For 16 the divisors are $\{1, 2, 4, 8, 16\}$ and for 35 the divisors are $\{1, 5, 7, 35\}$. The greatest divisor in common is GCD$(16, 35) = 1.$

12. Multiples of 10 are $\{10, 20, 30, 40, 50, 60, 70, 80, \ldots\}$ while the multiples of 12 are $\{12, 24, 36, 48, 60, 72, 84, \ldots\}$. The smallest multiple in common is LCM$(10, 12) = 60$.

13. Using the fact that GCD$(a, b) \cdot$ LCM$(a, b) = a \cdot b$, we have $15 \cdot$ LCM$(a, b) = 390$. Dividing both sides by 15 gives LCM$(a, b) = 26$.