Math 3120
Homework 24: Equivalent Sets, Finite Sets (5.1)
Due Friday, November 17, by 5pm. Be sure to show your work and justify your answers.

I encourage you to read the section, as you will be using previous results in some of your proofs.

A. Give a detailed outline of how to prove a function $f : A \to B$ is a bijection.
B. Define equivalent sets and $\mathbb{N}_k$.
C. Are the following sets equivalent? If they are, describe a bijection.
   a. $\mathbb{N}_7$ and the days of the week
   b. $(0, \infty)$ and $(-\infty, 0)$
   c. $(0, \infty)$ and $(0,1)$
   d. $\mathbb{N}$ and $\mathbb{Z}$
   e. $\mathbb{R}$ and $\mathbb{Z}$
   f. set of US governors and the set of states
   g. $\{0,4,8,...100\}$ and the set of letters in the alphabet
   h. $(0, 1)$ and $[0, 1]$ (take a guess!)
D. If two sets are equal, are they equivalent?
E. If two sets are equivalent, are they equal?
F. Give a bijection
   a. from the positive even numbers to $\mathbb{N}$
   b. from $\mathbb{N}$ to the positive even numbers
   c. from $\{a,e,i,o,u\}$ to $\{1,2,3,4,5\}$
   d. from $[0,1]$ to $[2,4]$
G. Prove $\approx$ is an equivalence relation on sets. I suggest using Theorem 4.12 (pg 203).

3b pg 228 If you use construction, be sure to prove your function is a bijection.

H. Define finite, infinite, and the cardinal number of a finite set.
I. What's wrong with the following pair of definitions?
   A finite set is a set that is not infinite.
   An infinite set is a set that is not finite.

5acd pg 228

J. Say whether the following sets are finite or infinite. If a set is finite, give the cardinality.
   a. $\{1,3,6,9\}$
   b. $\{3,4,5,...53\}$
   c. set of students in this class that usually come
   d. set of months with 30 days
   e. $\mathbb{N}_k$
   f. the set of prime numbers
   g. $\mathbb{R}$
   h. $\varnothing$
   i. $\{(x,y) \text{ in } \mathbb{R} \times \mathbb{R} \mid x^2 + y^2 = 1\}$
   j. $\{(x,y) \text{ in } \mathbb{R} \times \mathbb{R} \mid x^2 + y^2 = 0\}$
   k. $\{(x,y) \text{ in } \mathbb{R} \times \mathbb{R} \mid x^2 + y^2 = -1\}$
K. What kind of induction is used in the proof of Lemma 5.5 (pg 225)? Choose from weak, strong, or generalized.
L. Where in the proof of Theorem 5.7a (pg 226) do we use that A and B are disjoint?
M. Give the contrapositive of
   a. Theorem 5.7c (pg 226)
   b. Corollary 5.10 (pg 227)
N. What does Corollary usually signify?
O. What does Lemma often signify?

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P. How many elements are in $\mathbb{N}_k \times \mathbb{N}_m$?

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11ad pg 229

Q. In Theorem 5.2b (pg 223), the pairs of sets must be disjoint. Give a counterexample where at least one pair is not disjoint and the theorem does not hold.

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14 pg 229

17a pg 229 What have you really proved about f?

19d pg 229

21a pg 229

Math 3120
Homework 25: Infinite Sets (5.2)
Due Monday, November 20, by 5pm. Be sure to show your work and justify your answers.

Extra Credit Opportunity: Do corrections on Chapters 3 and 4 Test for half the points you missed back. Due the Monday after Thanksgiving by 5pm. All missed problems must be corrected or no extra points will be received. Be sure to have me look them over before the deadline, because there are no corrections accepted after the due date, even if you just had a tiny little thing to fix.

A. Define denumerable, $\aleph_o$, countable, uncountable, countably infinite, cardinality c (continuum).
B.
C. Read and understand the proof of Theorem 5.14 pg 233.
D. Read and understand the first example on pg 202. It is used to prove that $\mathbb{N} \times \mathbb{N}$ is denumerable.

1acd
2bf
3
5bdfg
6ac
7
8a
10
11
12be
Bonus: Prove the set of irrational numbers is infinite.

Math 3120
Homework 26: Countable Sets (5.3)
Due Monday, November 27, by 5pm. Be sure to show your work and justify your answers.

A. What's wrong with $w \in f(x)$?
B. What's the difference between f(x) and f(X)?
C. What is tacitly being used in the proof of Theorem 5.23?
D. Read about the Infinite Hotel (pg 239-230).
E. Outline briefly how to prove a set is countable.
F. Prove Theorem 5.18 pg 240.
G. Prove Lemma 5.27

1
2
8a
10
12ab
13a
15 (see Theorem 5.13)
16bcd

Math 3120
Homework 27: Ordering Cardinal Numbers (5.4)
Due Wednesday, November 29, by 5pm. Be sure to show your work and justify your answers.

A. What is the smallest cardinal number? What is the largest?
B. Read the proof of Theorem 5.31 pg 248.
C. Look up Georg Cantor online. Write down something interesting you learned about him. Be sure to cite your source(s).
D. Is $\leq$ a partial order on cardinal numbers? Justify.
E. Prove Theorem 5.30ab pg 248

1
2
3
5
7
8b
10
11c
17a

Bonus: Carefully read the proof of the Cantor-Schroder-Bernstein Theorem (pg 249-251).

Math 3120
Homework 28: Axiom of Choice (5.5)
Due Friday, December 1, by 5pm. Be sure to show your work and justify your answers.

A. Define axiom and theorem.
B. What is meant by Zermelo-Fraenkel set theory? Read the third paragraph on pg 70 and skim the axioms at http://en.wikipedia.org/wiki/Zermelo-Fraenkel_set_theory#The_axioms.
C. Give the Axiom of Choice in both mathematical terms and in your own words.
D. Where is the Axiom of Choice used in the proof of Theorem 5.22 pg 256?
E. Is $\aleph_o$ the smallest infinite cardinal number?
F. What is the continuum hypothesis?
G. Suppose a calculus student finding $\int x^3 dx$ gives the answer as $\frac{x^4}{4}$. What is the cardinality of the set of solutions the student has omitted?
H. Browse through some of the statements that the Axiom of Choice is equivalent to at http://en.wikipedia.org/wiki/Axiom_of_choice#Equivalents. Don't worry about understanding them.
I. What is meant when we say that the Axiom of Choice is equivalent to Zorn's Lemma?

1abde
3
4
5
6
10de
Bonus: Construct a bijection from (0,1] to (0,1). The hint from #8 might help.

Review Friday, Chapter 5 Test Monday, Review for the final Wednesday and Friday