Math 4410 Advanced Calculus II

HW 2

Due Friday, January 13, 2006, by 5pm.

1. Prove that if a sequence converges uniformly to a function, it also converges pointwise.
2. Give two examples of functions that are not differentiable.
3. Suppose $f(x) \equiv \lim_{n\to\infty} f_n(x)$ and the sequence converges uniformly. Also suppose $f(x)$ and $f_n(x)$ are differentiable.
   a. Does $f_n'(x)$ converge pointwise?
   b. If $f_n'(x)$ converges uniformly, does it converge to $f'(x)$?

   If the answer is yes, give a theorem to support your answer. Otherwise, give a counterexample.
4. Give the Taylor series for cos(x).
   a. Differentiate the series and compare the result to the derivative of cos(x).
   b. Integrate the series and compare the result to the integral of cos(x).
5. Summarize Theorems 8.14, 8.15, and 8.16 in your own words.
6. Let $f_k(x) = x^n$ and $F(x) = \sum_{n=1}^{\infty} x^n$.
   a. Give a formula for $F(x)$ and any necessary restrictions on the domain.
   b. Find $\int_0^{0.5} \sum_{n=1}^{\infty} x^n dx$ using the above formula.
   c. Find $\sum_{n=1}^{\infty} \int_0^{0.5} x^n dx$
   d. Repeat steps b & c with 0.5 replaced with 1.
   e. Relate your results in b, c, and d to Theorem 8.14.
7. Do 8.2 #10.
8. Read pages 245-252. Put a check to show you did it.