

Math 4410 Advanced Calculus II

Today: Topology

Next time: More Topology

HW 9

Due Thursday, February 2, 2006, by 10am.

1. Define topology.
2. Using the Euclidean norm, draw open and closed balls of radius 1 centered at the origin in 1, 2, and 3 dimensions.
3. Memorize the definitions of interior point, open set, boundary point, closed set, and closure of a set. You will be quizzed on Wed. as part of this homework assignment. ☺
4. For each of the following sets, state if it is open or closed and give its interior, boundary, and closure.
 - a. $(-1, 1]$
 - b. $\{x \mid x = 1/n, n \text{ is a natural number}\}$
 - c. natural numbers
 - d. integers
 - e. rationals
 - f. irrationals
 - g. reals
 - h. $\{(x, 0) \mid x \text{ is a real number}\}$
5. True or False? If false, give a counterexample. If true, cite a theorem.
 - a. A finite union of open sets is open.
 - b. An infinite union of open sets is open.
 - c. A finite union of closed sets is closed.
 - d. An infinite union of closed sets is closed.
 - e. A finite intersection of open sets is open.
 - f. An infinite intersection of open sets is open.
 - g. A finite intersection of closed sets is closed.
 - h. An infinite intersection of closed sets is closed.
 - i. A set cannot be both open and closed.
 - j. The intersection of a closed set and an open set can never be closed.
 - k. The intersection of a closed set and an open set can never be open.
 - l. A set that is not open is closed.
 - m. The closure of a set contains more points than the set.
 - n. A set has at least one interior point.
 - o. The boundary of a set contains at least one point.
 - p. An open ball is open.
 - q. The closure of an open ball is a closed ball.
6. How would you prove a set is not open? In other words, give the negation of open set.
7. How would you prove a set is not closed? In other words, give the negation of closed set.
8. If possible, give a function that for proper subsets of the real numbers ...
 - a. sends open sets to open sets
 - b. sends closed sets to closed sets
 - c. sends an open set to a closed set
 - d. sends a closed set to an open set
9. Do 9.2 #3a.