

General strategy for using the definition of a limit to prove $\lim_{x \rightarrow a} f(x) = L$

- Off to the side or on scratch paper write $|f(x) - L| < \varepsilon$.
- Manipulate until it looks like $|x - a| < \textit{something}$.
- Set $\delta = \textit{something}$.

Recall the definition of $\lim_{x \rightarrow a} f(x) = L$:

For every $\varepsilon > 0$, there exists a $\delta > 0$ such that if $0 < |x - a| < \delta$, then $|f(x) - L| < \varepsilon$.

- Start the proof by writing
Pf: Let $\varepsilon > 0$. Let $\delta = \textit{something}$. Then $0 < |x - a| < \delta \rightarrow \dots$
(here put the manipulations you used to get from $|f(x) - L| < \varepsilon$ to $|x - a| < \delta$ in reverse order.
The last thing you write should be $|f(x) - L| < \varepsilon$.)
- End the proof by writing
Thus $\lim_{x \rightarrow a} f(x) = L$. ■