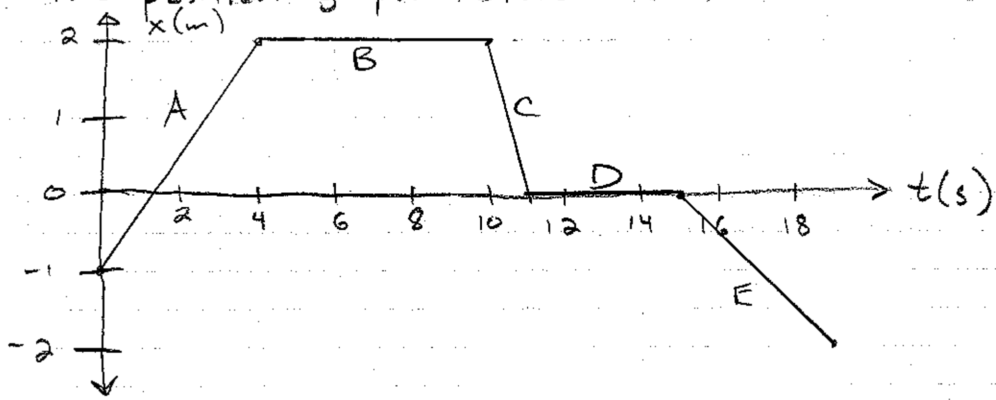
1 | Problem Set #2 Solutions

Conceptual: 9, 11, 14

(2) 9. The position graph referenced is:



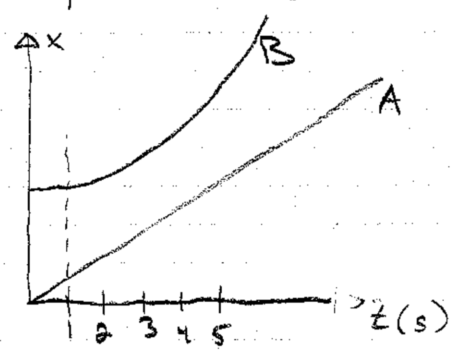
a) I walk 3 m forward to a blackboard, where I spend 7 seconds writing before suddenly taking a 2m step backwards. I spend 5 seconds stationary, reading what is written there before turning and strolling away from the board.

b) Object is at rest in segments <u>B & D</u> (constant position)

c) The object is moving to the right (the + direction) only in segment <u>A</u>

d) As the slope in C is steeper than in E (a more negative number), the speed in C will be <u>greater than</u> the speed in E.

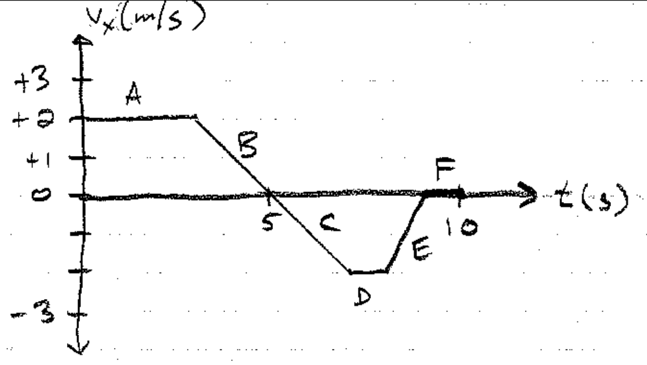
(2) 11.



a) At $t = 1$ s, the slope of B is less than the slope of A, so the speed of A is <u>greater than</u> the speed of B.

b) The objects have the same speed when they have the same slope which occurs only around <u>$t = 3$ s</u> (and only at that instant

(2) 14.



a) Velocity is constant in segments <u>A</u>, <u>D</u>, <u>F</u>

b) Object is speeding up in segment <u>C</u>

c) Object is slowing down in segments <u>B</u> and <u>E</u>

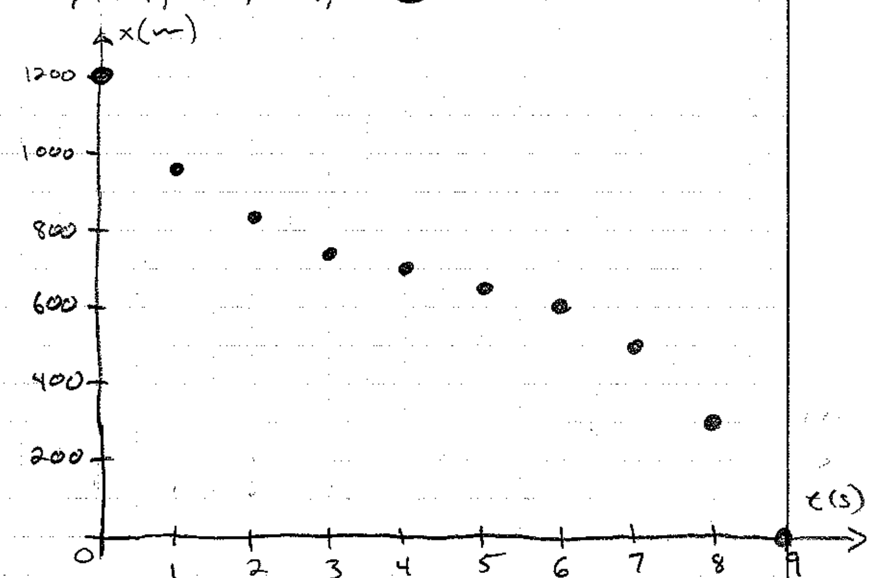
d) Object is standing still in segment <u>F</u>

e) Object is moving to the right (+) in segments <u>A</u> and <u>B</u>

Problems: 1, 6, 9, 13, 19, 22, 29, 30, 49, 70

(2) 1. a)

x(m)	t(s)
1200	0
970	1
830	2
740	3
700	4
650	5
600	6
500	7
300	8
0	9



(we don't connect dots here).

(2) 6. Car moves w/constant $\vec{v} = (10.\ m/s, NE)$; speed $v = 10.\ m/s$

After a time $\Delta t = 45\ s$; car will be $\Delta x = v\Delta t = \boxed{450.\ m}$

away from origin

③

(6) 9. Richard drives a distance of 125 mi. Given a speed limit of 65 mph; it would normally take him:

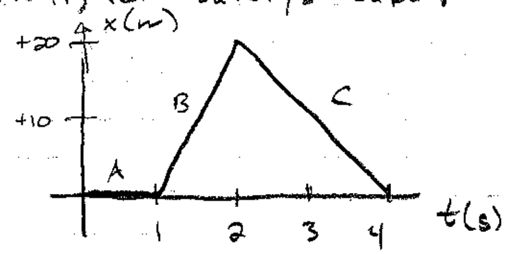
$$\Delta x = v\Delta t \Rightarrow \Delta t_1 = \frac{\Delta x}{v_1} = \frac{125 \text{ mi}}{65 \text{ mi/hr}} = 1.92 \text{ hours, or } 1 \text{ h}, 55 \text{ m}$$

Recklessly driving 70 mph instead, Richard's new time is

$$\Delta t_2 = \frac{\Delta x}{v_2} = \frac{125 \text{ mi}}{70 \text{ mi/hr}} = 1.79 \text{ hours, or } 1 \text{ h}, 47 \text{ m}$$

So, he saves $\Delta t_2 - \Delta t_1 = \boxed{8 \text{ minutes}}$ on the trip. But was it really worth it, ~~for~~ safety's sake?

(4) 13. Position Graph

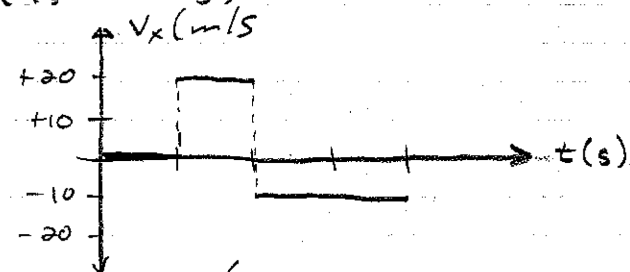


a)

in A: $v_x = \frac{\Delta x}{\Delta t} = 0. \text{ m/s}$

in B: $v_x = \frac{\Delta x}{\Delta t} = \frac{(20 \text{ m} - 0 \text{ m})}{(2\text{ s} - 1\text{ s})} = +20 \text{ m/s}$

in C: $v_x = \frac{\Delta x}{\Delta t} = \frac{(0\text{m} - 20.\text{m})}{(4\text{s} - 2\text{s})} = -10 \text{ m/s}$



b) Object has a turning point (+ → − or − → + velocity) at $\boxed{t = 2\text{s}}$

(4) 19. We estimate numerical values from the graph.

in A: $a_x = \frac{\Delta V_x}{\Delta t} = \frac{(5.5\,m/s - 0\,m/s)}{(0.9s - 0\,s)} = \boxed{6.1\ m/s^2}$

in B: $a_x = \frac{\Delta V_x}{\Delta t} = \frac{(9.2\,m/s - 5.5\,m/s)}{(2.4s - 0.9s)} = \boxed{2.5\ m/s^2}$

in C: $a_x = \frac{\Delta V_x}{\Delta t} = \frac{(10.8\,m/s - 9.2\,m/s)}{(3.3s - 2.4s)} = \boxed{1.8\ m/s^2}$

(Note: my answer differs slightly from the answer in the back, due to estimating by eye from the graph).

(6) 22. a) 0 to 60 mph in 10 s.

$a = \frac{\Delta V}{\Delta t} = \frac{(60\,mph - 0\,mph)}{10\,s}\left(\frac{.447\,m/s}{1\,mph}\right) = \boxed{2.68\ m/s^2}$

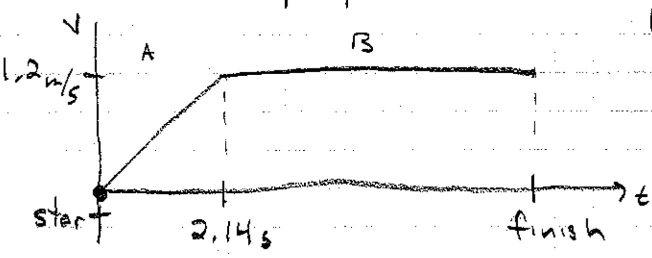
b) this is: $\frac{a}{g} = \frac{2.68\ m/s^2}{9.81\ m/s^2} = \boxed{.27}$, or $\frac{27}{100}$ g (or ~ $\frac{1}{3}$ g)

c) $\Delta t = 10.\ s$ use position equation
$V_i = 0$

$\therefore\ \Delta X = (0)(10.\,s) + \frac{1}{2}(2.68\,m/s^2)(10\,s)^2 = \boxed{134\ m}$

$= 134\,m\left(\frac{39.37\,in}{1\,m}\right)\left(\frac{1\ foot}{12\ in}\right) = \boxed{440.\ ft}$

(4) 29. $\Delta X = 100.\ m$
reaches top speed of 11.2 m/s in 2.14 s. We want to know the total time from start to finish.



in segment A: Runner has moved a displacement ΔX_A

$V_i = 0$ $x_i = 0$
$V_f = 11.2\ m/s$ $x_f = ?$
$\Delta t_A = 2.14\ s$ $a = ?$

Use velocity equation to find a: $11.2\ m/s = 0\ m/s + a(2.14\,s)$

$a = 5.234\ m/s^2$

Then velocity displacement equation to find ΔX_A

29. (continued)

$$(11.2\,m/s)^2 = (\phi\,m/s)^2 + 2(5.234\,m/s^2)(\Delta x_A)$$

$\Rightarrow \Delta x_A = 11.984\ m$ (hold onto sigfigs for now.)

Thus, in part B, Runner moves $(\Delta x_B = 100.\,m - \Delta x_A) = 88.016\,m$ at a <u>constant</u> velocity of $11.2\,m/s$:

$$\Delta x_B = v\,\Delta t_B \Rightarrow \Delta t_B = \frac{88.016\,m}{11.2\,m/s} = 7.86\ s.$$

$\therefore$ the total time $\Delta t_A + \Delta t_B = 2.14\,s + 7.86\,s = \boxed{10.00\,s}$

(6) 30. Ball bearings falling through a shot tower.

a) If they need $\Delta t = 4.0\,s$ (and assume they fall from rest)

then $\Delta y = (\phi)\Delta t + \frac{1}{2}a_y(\Delta t)^2$

$$= -\tfrac{1}{2}g(\Delta t)^2 = -78\ m \text{ on way down,}$$

Tower needs to be at least $\boxed{78\,m}$ high.

b) Their impact velocity is the velocity just before impact

$$V_{yf} = V_{yi} + a_y\Delta t = -g\cdot\Delta t = \boxed{-39\ m/s}$$

(4) 49. a) $c = 3.0\times10^8\,m/s$ = speed of light.

Acceleration from rest at $a = g$

$$V_f = c = v_i + a\Delta t = \phi + g\cdot\Delta t$$

$$\Rightarrow \Delta t = \frac{c}{g} = 3.06\times10^7\,s\left(\frac{1\ day}{86400\,s}\right) = \boxed{354\ days}$$

(~ year)

b) $V_f^2 = c^2 = v_i^2 + 2a\Delta x = \phi + 2g\Delta x$

$$\Delta x = \frac{c^2}{2g} = \boxed{4.6\times10^{15}\,m}$$

c) A light-year $= c\,(1\ year) = (3.0\times10^8\,m/s)(365.25\,days)\left(\frac{86400\,s}{1\,day}\right)$

$$= 9.5\times10^{15}\,m$$

So you've gone $\frac{4.6\times10^{15}\,m}{9.5\times10^{15}\,m} \simeq$ $\boxed{0.5\text{, or half a light year}}$

(8) 70. Here we split up the motion into two segments.

A: Rocket (+ bolt) from liftoff (rest, ground level) to when the bolt detaches 4.0 s later (unknown height, unknown velocity).

B: Bolt from when it detaches to when it hits the ground (ground level) after free-falling for 6.0 s.

<u>Both</u> segments are const. acceleration, but with different values.

Set up variables for both:

A) $y_{Ai} = 0$	$v_{Ai} = 0$	B) $y_{Bi} = y_{Af} = ?$	$v_{Bi} = v_{Af} = ?$
$y_{Af} = ?$	$v_{Af} = ?$	$y_{Bf} = 0$	$v_{Bf} = ?$
$\Delta t_A = 4.0\,s$	$a_y = ? = a_A$	$\Delta t_B = 6.0\,s$	$a_y = -g$

Note: final values of segment A are initial values of segment B.

What we'll have to do is set-up a system of equations that will enable us to solve for our unknown accelerations.

A) pos. eqn. $y_{Af} = 0 + 0\,\Delta t + \frac{1}{2} a_A (\Delta t_A)^2$

or

$$y_{Af} = \frac{1}{2} a_A (\Delta t_A)^2$$

vel. eqn. $v_{Af} = 0 + a_A \Delta t_A$

or

$$v_{Af} = a_A \Delta t_A$$

So, while we don't yet know y_{Af} or v_{Af}, we've written them in terms of a single unknown (a_A).

B) pos. eqn. $y_{Bf} = y_{Bi} + v_{Bi} \Delta t_B + \frac{1}{2}(-g)(\Delta t_B)^2$

$$0 = y_{Af} + v_{Af} \Delta t_B - \frac{1}{2} g (\Delta t_B)^2$$

Now, insert the expressions for y_{Af} and v_{Af}

70. (cont)

$$0 = \frac{1}{2} a_A (\Delta t_A)^2 + (a_A \Delta t_A) \Delta t_B - \frac{1}{2} g (\Delta t_B)^2$$

Now we have a linear equation in a_A we can solve

$$0 = a_A \left[\frac{1}{2} (\Delta t_A)^2 + \Delta t_A \Delta t_B \right] - \frac{1}{2} g (\Delta t_B)^2$$

$$\frac{1}{2} g (\Delta t_B)^2 = a_A \left[\frac{1}{2} (\Delta t_A)^2 + \Delta t_A \Delta t_B \right]$$

$$a_A = \frac{\left(\frac{1}{2} g (\Delta t_B)^2 \right)}{\frac{1}{2} (\Delta t_A)^2 + \Delta t_A \Delta t_B}$$

$$a_A = \frac{\frac{1}{2} (9.81 \, m/s^2)(6.0 \, s)^2}{\frac{1}{2} (4.0 \, s)^2 + (4.0 s)(6.0 s)}$$

$$\boxed{a_A = +5.5 \, m/s^2}$$

Note: while we weren't asked to, we could now compute the height where the bolt broke off:

$$y_{Af} = \frac{1}{2} a_A (\Delta t_A)^2 = 44. \, m$$

and the velocity it had when it broke off

$$v_{Af} = a_A \Delta t_A = +22 \, m/s$$