1. Notes about the examination:
   - The examination should take about about an hour and a half.
   - The exam is entirely show–your–work.
   - The exam will be taken in the Testing Center [please go here for more information], and will be open Thursday, February 2 and Friday, February 3.
   - You will have a choice of doing 10 out of 13 or so problems (worth 10 points each). You must **SHOW YOUR WORK** where appropriate to receive full credit for a problem.
   - Please do all of your work on the white paper provided – the only thing that should be on the examination paper itself is your name.
   - **No cell–phone calculators** will be allowed in the Testing Center!
   - Good luck (if you are depending on luck)!

2. Definitions:
   set, element $\in$, empty set $\emptyset$, well–defined, subset $\subseteq$, improper, proper $\subset$, Cartesian product $\times$, relation $\mathcal{R}$ on S, function, domain, codomain, range, cardinality $|S|$, one–to–one, onto, inverse function f^{-1}, infinite set, disjoint, partition, residue class modulo n, equivalence relation, congruence modulo n, equivalence class, power set $\mathcal{P}(X)$, complex numbers, imaginary number, absolute value, Euler's Formula, addition modulo 2π, n^{th} roots of unity, binary operation $*$, closed, commutative, associative, group table, well–defined, idempotents, algebraic (binary) structure, isomorphism ϕ, homomorphism, structural property, identity e, inverse x^{-1}, semigroup, monoid, group, abelian, general linear group $GL_n(\mathbb{R})$, Cancellation Laws, Unique Solutions, *Socks and Shoes*, finite group tables, *Sudoku rules*, diagonal matrix, main diagonal, upper–triangular matrix, order $|G|$ of a group G, subgroup $H \leq G$, trivial subgroup, proper subgroup, Klein vier group V, subgroup diagram, Two–Step Subgroup Test, cyclic group, generator, cyclic subgroup $\langle g \rangle$ generated by g, orthogonal matrices, transpose A^T, center of G, order $|g|$ of a group element g, Division Algorithm, quotient q, remainder r, greatest common divisor gcd, relatively prime, automorphism, primitive n^{th} roots of unity, least common multiple lcm

3. Chapter 0 stuff...
   ex: Prove that the composition of functions is again a function
   ex: Show that the relation $\equiv_n$ defined on the integers $\mathbb{Z}$ as $x \equiv_n y \Leftrightarrow n$ divides $x - y$ is an equivalence relation (for each fixed natural number n).

4. Write a complex number in its polar form
   ex: $15 - 8i$
   ex: Using the definitions on p. 20, show that $e^{i\theta} = \cos(\theta) + \sin(\theta)i$.

5. Determine if a function (on a set) represents a binary operation
   ex: Define $f\colon \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ as $f(r_1, r_2) = \sqrt{r_1 r_2}$
   ex: Define $g\colon \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ as $g(r_1, r_2) = r_1^{r_2}$

6. Determine which of the 5 properties (closed, associative, identity, inverse, commutative) a binary operation has

   ex: For $\{x, y\} \subseteq \mathbb{N}$, let $\ominus$ be defined as $x \ominus y = 2x + 3y$

   ex: For $\{X, Y\} \subseteq M_4(\mathbb{R})$, let $\boxplus$ be defined as $X \boxplus Y = XY$

   ex: For $\{[x], [y]\} \subseteq \mathbb{Z}_8$, let $\Diamond$ be defined as $[x] \Diamond [y] = [x + y - xy]$

7. Write out the group table for an algebraic structure

   ex: Let $X = \langle \{1, 3, 5, 7\}, \times_8 \rangle$. Find its group table

8. Determine if a function is a homomorphism

   ex: Define $h\colon \langle \mathbb{C}, \times \rangle \to \langle \mathbb{R}, \times \rangle$ as $h(c) = |\, c \,|$

   ex: Define $j\colon \langle 6\mathbb{Z}, + \rangle \to \langle 15\mathbb{Z}, + \rangle$ as $j(z) = 5z$

9. Determine if a function is an isomorphism

   ex: Define $\phi\colon \langle \mathbb{Z}, + \rangle \to \langle 7\mathbb{Z}, + \rangle$ as $\phi(x) = 7x$

   ex: Define $\psi\colon \langle \mathbb{Z}_6, +_6 \rangle \to \langle \mathbb{Z}_3, +_3 \rangle$ as $\psi(z) = z$

10. Determine whether a property is a structural property or not

    ex: For an algebraic system with identity e, the number of solutions to the equation $x^2 = e$.

11. Determine if a set together with a binary operation determines a group

    ex: Let $X = \{1, 5, 7, 11\}$. Is $\langle X, \times_{12} \rangle$ a group? Is it abelian?

    ex: Show that the set $\mathbb{Z}_5^*$ together with $\times_5$ is an abelian group

    ex: In a group G, show that $a^{-1}b^{-1}c^{-1} = (cba)^{-1}$ for all $\{a, b, c\} \subseteq G$

12. Determine if a subset is a subgroup (usually by the *Two–Step Subgroup Test*)

    ex: Let G be a group (with identity e) and let $a \in G$. Prove that the set $C_a = \{g \in G : ga = ag\}$ is a subgroup of G. (This is called the **centralizer** of a in G.)

    ex: Show that for any group G, the set G itself forms a subgroup.

13. Determine if a group is cyclic

    ex: $\mathbb{Z}_7^*$ with respect to $\times_7$ (Follow up question: How many generators does it have?)

    ex: Show that every subgroup of a cyclic group is cyclic
    (*Remember* – the proof requires the *Division Algorithm*)

14. Determine a cyclic subgroup generated by an element

    ex: In the group $\langle \mathbb{C}^*, \times \rangle$, what is $\langle \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i \rangle$?

    ex: What is the order of the element ζ^8 in the group U_{20}?

15. Find all generators of a cyclic group

    ex: The subgroup $\langle 6 \rangle$ in the group $\mathbb{Z}_{40}$.