Name (5 points): Key

Section I

Answer any 3 of problems 1-4. Please circle the 3 question numbers that you want me count, and cross out the one you don't want me to count (if you don't I'll count 1-3). Each of the three questions you choose to count will be worth 10 points.

1) Write the Lagrangian of the problem of maximizing utility from playing tennis and riding your bike, subject to a constraint on time. You can assume a utility function like the one used in class, but it should show that you like playing tennis more than riding your bike. You may also assume that you have 4 hours per week to play tennis or ride your bike, and that each tennis match or bike ride costs one hour.

$\mathcal{L} = T^{\alpha} R^{\beta} + \lambda(4 - T - R)$; with $\alpha > \beta$

2) After setting up a Lagrangian, every constrained optimization problem is solved in three steps. The first step is to: ________________.

Take partial derivatives

3) After setting up a Lagrangian, every constrained optimization problem is solved in three steps. Why is the second step to set the partial derivatives to zero?

A maximum or minimum must be where slopes (partial derivatives) are zero.

4) After setting up a Lagrangian, every constrained optimization problem is solved in three steps. The third step is to: ________________.

Solve the system of equations

Section II

Answer any 3 of problems 5-9. Please circle the 3 question numbers that you want me count, and cross out the one you don't want me to count (if you don't I'll count 5-7). Each of the three questions you choose to count will be worth 20 points.

5) Show where this function, $f(x) = -x^3 + 12x + 7$, has optima, and then determine if each is a maximum or a minimum.

$$f' = -3x^2 + 12 = 0$$

$$3x^2 = 12$$

$$x^2 = 4$$

$$x = \pm 2$$

$$f'' = -6x \qquad f''(2) = -12 \Rightarrow \text{max at } 2$$

$$f''(-2) = 12 \Rightarrow \text{min at } -2$$

6) Your construction company builds houses on rectangular lots. Corner lots require a block wall on only 2 adjacent sides out of the four. Is the lot that maximizes area subject to a constraint that the length of the block wall be 16, square? You may use either substitution or the Lagrangian method.

$$\mathcal{L} = WL + \lambda(16 - W - L)$$

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial W} &= L - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial L} &= W - \lambda = 0 \end{aligned}\right\} \quad \frac{L}{W} = 1 \Rightarrow L = W$$

$\therefore$ a square!

Name (5 points): Key

Section II Continued

Answer any 3 of problems 5-9. Please circle the 3 question numbers that you want me count, and cross out the one you don't want me to count (if you don't I'll count 5-7). Each of the three questions you choose to count will be worth 20 points.

7) Use Solver to maximize production of markers, where $Q = \ln(L) + 2\ln(K)$. How many markers will you make if you have \$3000 to spend, and labor costs \$1, while capital costs \$2? How many more markers will you make if your budget goes up by a dollar? Submission of a spreadsheet through e-mail is only required if you want a chance at partial credit. See the spreadsheet key too.

$Q = 20.7$

$\lambda = .001$

8) Consider the equation $\frac{Q_x^{-0.5}}{Q_y^{-0.5}} = \frac{6}{11}$. Could this equation be showing that the marginal rate of substitution equals the marginal rate of transformation? (Hint: does it behave like that part of a constrained optimization problem).

Yes it could. If so, the left side is the MRS and the right side the MRT (the ratio of prices). You need to show that this behaves the right way:

$$P_x \uparrow \Rightarrow \frac{Q_x^{-0.5}}{Q_y^{-0.5}} = \sqrt{\frac{Q_y}{Q_x}} \uparrow \Rightarrow Q_x \downarrow \checkmark$$

$$" \quad " \quad " \Rightarrow Q_y \uparrow \checkmark$$

Section II Continued

Answer any 3 of problems 5-9. Please circle the 3 question numbers that you want me count, and cross out the one you don't want me to count (if you don't I'll count 5-7). Each of the three questions you choose to count will be worth 20 points.

9) How many rooms and professors will a college employ to maximize the production of Q classes with R rooms and P professors. Assume that production is given by $Q = R^{\alpha}P^{(1-\alpha)}$, that there are M dollars to spend, and that rooms and professor each cost 1.

$$\mathcal{L} = R^{\alpha}P^{(1-\alpha)} + \lambda(M - R - P)$$

$$\frac{\partial \mathcal{L}}{\partial R} = \alpha R^{\alpha-1}P^{1-\alpha} - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial P} = (1-\alpha)R^{\alpha}P^{-\alpha} - \lambda = 0$$

$$\frac{\alpha R^{\alpha-1}P^{1-\alpha}}{(1-\alpha)R^{\alpha}P^{-\alpha}} = \frac{\lambda}{\lambda}$$

$$\frac{\alpha P}{(1-\alpha)R} = 1$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = M - R - P = 0$$

$$R = \frac{\alpha P}{1-\alpha}$$

$$M = \frac{\alpha P}{1-\alpha} + P$$

$$= \left(\frac{\alpha}{1-\alpha} + \frac{1-\alpha}{1-\alpha}\right)P$$

$$= \frac{1}{1-\alpha}P$$

$$P = (1-\alpha)M$$

$$R = \frac{\alpha P}{1-\alpha} = \frac{\alpha(1-\alpha)M}{(1-\alpha)} = \alpha M$$