ECON 2500-1 Exam 3 Key Fall 2009

Name (5 points):____

Section I

Answer any 3 of problems 1-4. Please circle the 3 question numbers that you want me count, and cross out the one you don't want me to count (if you don't I'll count 1-3). Each of the three questions you choose to count will be worth 10 points.

1) Write the Lagrangian of the problem of minimizing time spent studying and reading, subject to a grade constraint. You can assume a production function for grades like the one used in class, but it should show that reading is more important to producing grades than is studying. You may also assume that your minimum target grade is a 90, and that each studying or reading period costs you one hour.

$$\mathcal{L} = T_S + T_R + \lambda\left(90 - T_S^{1-\alpha} T_R^{\alpha}\right), \text{ for } 1 > \alpha > 0.5$$

2) After setting up a Lagrangian, every constrained optimization problem is solved in three steps. Why is the first step taking partial derivatives?

These are the slopes in different directions of the function being optimized

3) After setting up a Lagrangian, every constrained optimization problem is solved in three steps. The second step is to: ______________________________

Set the partial derivatives to zero.

4) After setting up a Lagrangian, every constrained optimization problem is solved in three steps. What is the third step and what function does it serve in solving the overall problem?

You have a system of equations in which the variables of interest are mixed. Solving that system allows you to express each of those variables independent of the other ones.

Section II

Answer any 3 of problems 5-9. Please circle the 3 question numbers that you want me count, and cross out the one you don't want me to count (if you don't I'll count 5-7). Each of the three questions you choose to count will be worth 20 points.

5) Show where this function, $f(x) = x^3 - 3x + 7$, has optima, and then determine if each is a maximum or a minimum.

$f' = 3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$

$f'' = 6x$

$f''(1) = 6 \Rightarrow 1$ is a ~~max~~ min

$f''(-1) = -6 \Rightarrow -1$ is a max

6) Your construction company builds houses. Block walls are required on three sides of the lot in the subdivision you will build in. Minimize the length of the block wall, subject to a constraint that the area of the lot be at least 5,000 square feet. You may use either substitution or the Lagrangian method.

$\min_{W,L} \; 2L + W + \lambda(5{,}000 - WL)$

$\mathcal{L} = 2L + W + \lambda(5{,}000 - WL)$

$\frac{\partial \mathcal{L}}{\partial L} = 2 - \lambda W = 0$

$\frac{\partial \mathcal{L}}{\partial W} = 1 - \lambda L = 0$

$\frac{\partial \mathcal{L}}{\partial \lambda} = 5{,}000 - WL = 0$

$\frac{2 = \lambda W}{1 = \lambda L} \Rightarrow 2L = W$

$5{,}000 = WL$

$= 2L^2$

$2{,}500 = L^2$

$50 = L$

$100 = W$

Name (5 points): Key

Section II Continued

Answer any 3 of problems 5-9. Please circle the 3 question numbers that you want me count, and cross out the one you don't want me to count (if you don't I'll count 5-7). Each of the three questions you choose to count will be worth 20 points.

7) Use Solver to maximize utility for pencils and markers, where $U = (Q_P^{0.4} + Q_M^{0.4})^{2.5}$. How many markers will you buy if you have \$30 to spend, and pencils cost \$1, while markers cost \$2? How many markers will you buy if the price of pencils goes up to \$2? Submission of a spreadsheet through e-mail is only required if you want partial credit. See the spreadsheet key too.

$Q_M(1) = 5.8$

$Q_M(2) = 7.5$

8) Consider the equation $\frac{1/Q_x}{2/Q_y} = \frac{6}{11}$. Could this equation be showing that the marginal rate of substitution equals the marginal rate of transformation? (Hint: does it behave like that part of a constrained optimization problem).

If this equation is MRS = MRT, then prices are on the right-hand-side. So 6 is the price of good X. Then increasing this price from 6 to 7 should lead to Q_x going down or Q_y going up.

$6 \uparrow \Rightarrow 1/Q_x \uparrow \Rightarrow Q_x \downarrow$ ✓

$6 \uparrow \Rightarrow 2/Q_y \downarrow \Rightarrow Q_y \uparrow$ ✓

Section II Continued

Answer any 3 of problems 5-9. Please circle the 3 question numbers that you want me count, and cross out the one you don't want me to count (if you don't I'll count 5-7). Each of the three questions you choose to count will be worth 20 points.

9) Minimize the costs of producing Q classes with R rooms and P professors. Assume that rooms and professor each cost 1, and that production is given by $Q = R^{\alpha} P^{(1-\alpha)}$.

$$\mathcal{L} = R + P + \lambda(Q - R^{\alpha}P^{1-\alpha})$$

$$\frac{\partial \mathcal{L}}{\partial R} = 1 - \lambda\alpha R^{\alpha-1}P^{1-\alpha} = 0$$

$$\frac{\partial \mathcal{L}}{\partial P} = 1 - \lambda(1-\alpha)R^{\alpha}P^{-\alpha} = 0$$

$$\frac{1}{1} = \frac{\lambda\alpha R^{\alpha-1}P^{1-\alpha}}{\lambda(1-\alpha)R^{\alpha}P^{-\alpha}}$$

$$1 = \frac{\alpha P}{(1-\alpha)R}$$

$$P = \frac{(1-\alpha)R}{\alpha}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Q - R^{\alpha}P^{1-\alpha} = 0$$

$$Q = R^{\alpha}P^{1-\alpha}$$

$$= R^{\alpha}\left[\frac{(1-\alpha)R}{\alpha}\right]^{1-\alpha}$$

$$= R\left[\frac{(1-\alpha)}{\alpha}\right]^{1-\alpha}$$

$$R = Q\left[\frac{(1-\alpha)}{\alpha}\right]^{\alpha-1}$$

$$P = Q\left[\frac{(1-\alpha)}{\alpha}\right]^{\alpha-1}\left[\frac{(1-\alpha)}{\alpha}\right] = Q\left[\frac{(1-\alpha)}{\alpha}\right]^{\alpha}$$